CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination 2
Secondary Four

MATHEMATICS **4016/02**

Paper 2 **16 August 2011**

2 hours 30 minutes

Additional Materials: Answer Paper
Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Given that $y = \dfrac{k - x^3}{\pi x^3 + 1}$, express x in terms of y, k and π. [2]

(b) Express as a single fraction in its simplest form $\dfrac{1}{2(x-2)^2} - \dfrac{3}{x(x-2)}$. [2]

(c) **(i)** Express $x^2 - \dfrac{3}{4}x - 5$ in the form $(x+a)^2 + b$. [1]

(ii) **Hence** solve the equation $x^2 - \dfrac{3}{4}x - 5 = 0$, giving your answers correct to two decimal places. [3]

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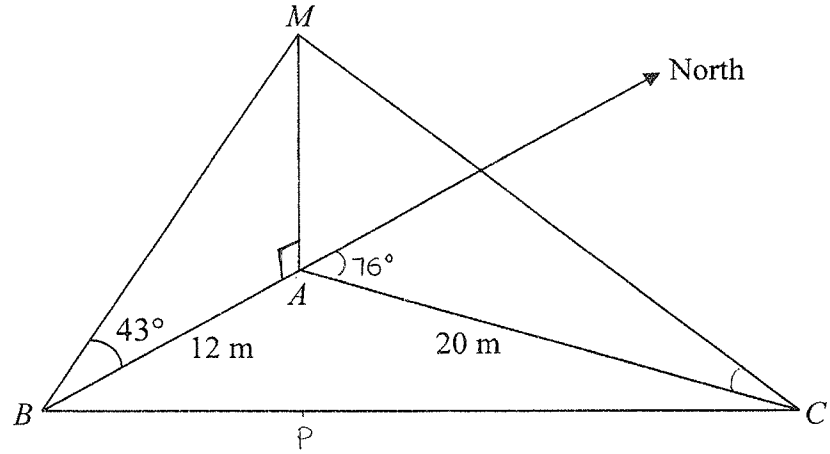
2 Mr Goh bought a home entertainment system during the Year-End Sale on hire purchase. He paid a deposit of 20% of the selling price. The balance of the payment was $8 000.

(a) Calculate the selling price of the system. [1]

(b) Mr Goh paid the balance of the payment in monthly instalments of $380, charged at x % per annum simple interest for 2 years. Find the value of x. [3]

(c) Mr Goh could have taken a loan to pay the outstanding balance. If the finance company charged him a compound interest of 6.5 % per annum with repayment period of 2 years, would you recommend that he takes up the loan instead of paying in instalments? State your reason clearly. [3]

3



Three points, A, B and C, lie on a horizontal field. A is due North of B and the bearing of C from A is $076°$. M is vertically above A. Angle $MBA = 43°$, $AB = 12$ m and $AC = 20$ m.

(a) Calculate

(i) MA, [2]

(ii) the angle of depression of C from M, [2]

(iii) BC, [2]

(iv) the area of triangle ABC. [2]

(b) P is a point on BC such that the angle of elevation of M from P is the greatest. Calculate the angle of elevation of M from P. [3]

4 A car travels x kilometres on each litre of petrol when it is driven on rough terrain. It can travel 2 more kilometres on each litre of petrol when it is driven on an expressway.

(a) Write down an expression, in terms of x, for the number of litres of petrol used when the car is driven on rough terrain for 400 km. [1]

(b) Write down an expression, in terms of x, for the number of litres of petrol used when the car is driven on an expressway for 400 km. [1]

(c) The car uses 5 litres less of petrol when it is driven on an expressway for 400 km compared to when it is driven on rough terrain. Write down an equation in x to represent this information, and show that it reduces to $x^2 + 2x - 160 = 0$. [3]

(d) Solve the equation $x^2 + 2x - 160 = 0$, giving both answers correct to two decimal places. [3]

(e) Calculate the number of litres of petrol used when the car is driven on an expressway for **100 km**. [2]

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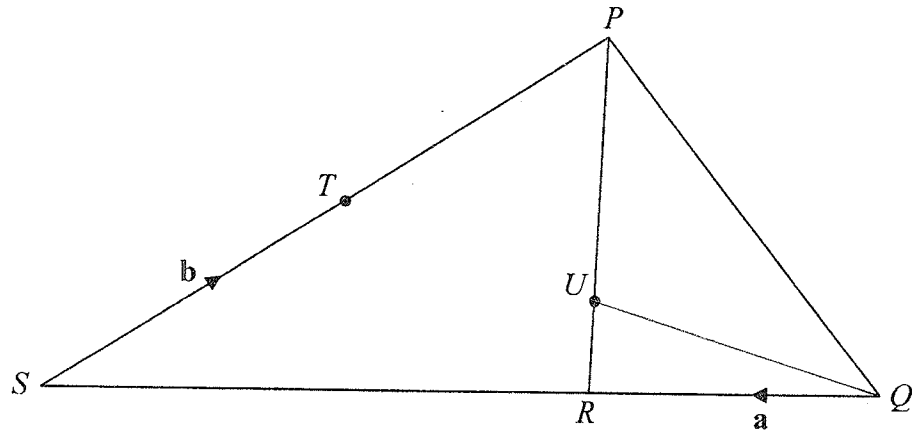
5 **(a)** It is given that $\overrightarrow{AB}$ is $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$.

(i) Calculate $\left|2\overrightarrow{AB}\right|$. [1]

(ii) Calculate the gradient of the line AB. [1]

(iii) Given that A is the point $(-5, 2)$, find the coordinates of B. [1]

(b)



In the diagram, $\overrightarrow{QR} = \mathbf{a}$ and $\overrightarrow{ST} = \mathbf{b}$. R is the point on QS such that $\overrightarrow{QS} = 3\overrightarrow{QR}$. U is the point on RP such that $\overrightarrow{RP} = 4\overrightarrow{RU}$. T is the midpoint of SP.

(i) Express, as simply as possible, in terms of **a** and/or **b**,

(a) $\overrightarrow{RS}$, [1]

(b) $\overrightarrow{RP}$, [1]

(c) $\overrightarrow{UQ}$, [2]

(d) $\overrightarrow{TQ}$. [1]

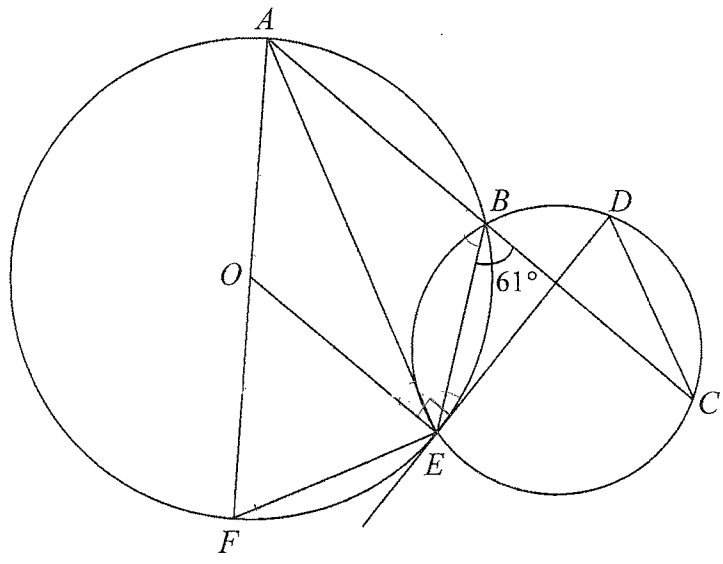
(ii) Write down two facts about the points T, U and Q. [2]

(iii) Calculate the value of

(a) $\dfrac{\text{area of triangle } PQU}{\text{area of triangle } PQR}$, [1]

(b) $\dfrac{\text{area of triangle } RUQ}{\text{area of triangle } PQS}$. [2]

6



In the diagram, DE is a tangent to the circle with centre O. ABC and AOF are straight lines and $\angle CBE = 61°$.

(a) Giving your reasons, find

(i) angle AOE, [2]

(ii) angle AFE, [2]

(iii) angle AEO, [2]

(iv) angle AED. [2]

(b) Are the lines AE and DC parallel? Give a reason for your answer. [2]

7 **(a)** A **four-sided** unbiased die and a **six-sided** unbiased die are thrown at the same time. The score of each die is noted.

(i) Draw a possibility diagram to show the outcomes of the throw. [1]

(ii) Find, as a fraction in its simplest form, the probability that

(a) the sum of the scores is 7, [1]

(b) the product of the scores is a multiple of 3. [1]

(b) Box *A* contains 3 cups of chocolate ice-cream and 5 cups of strawberry ice-cream. Box *B* contains 2 cups of chocolate ice-cream, 4 cups of strawberry ice-cream and 3 cups of vanilla ice-cream. A cup of ice-cream is selected at random from box *A*. It is then placed in box *B* before a cup of ice-cream is selected at random from box *B*.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

(ii) Find, as a fraction in its simplest form, the probability that

(a) the two cups of ice-cream selected are of the same flavour, [2]

(b) the second cup of ice-cream selected is strawberry. [2]

8 Ashley, Ben and Celia are salespersons at a travel agency. The sales that they obtained for each of the three tour packages in May and June are shown in the table below.

	May			June		
	Ashley	**Ben**	**Celia**	**Ashley**	**Ben**	**Celia**
Package 1	5	8	6	9	14	15
Package 2	7	4	5	13	10	9
Package 3	4	3	5	12	11	10

The information for the month of May can be represented by the matrix

$$\mathbf{M} = \begin{pmatrix} 5 & 8 & 6 \\ 7 & 4 & 5 \\ 4 & 3 & 5 \end{pmatrix}.$$

(a) The information for the month of June is represented by the matrix **J**.

(i) Write down the matrix **J**. [1]

(ii) Evaluate **J** – **M**.
State what the elements of **J** – **M** represent. [2]

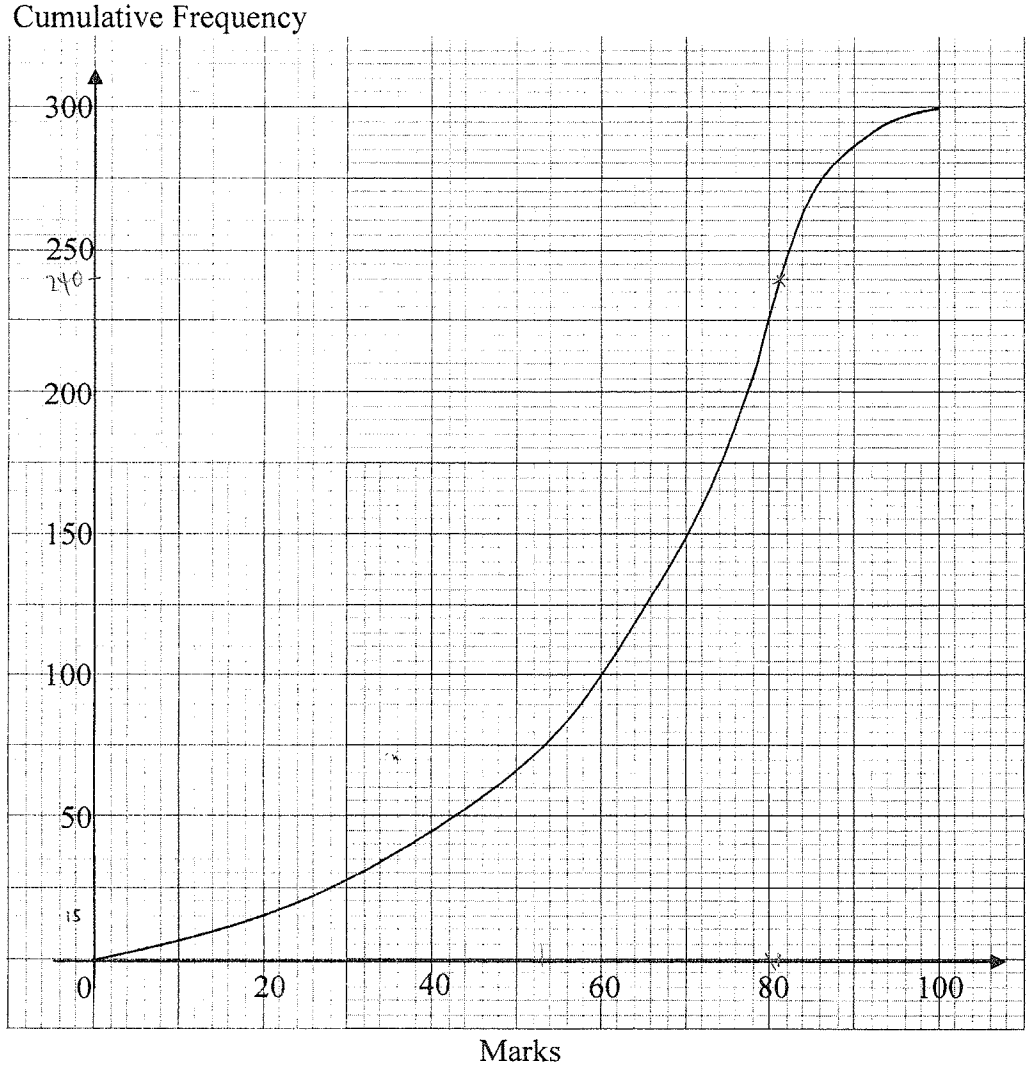
(b) Write down an appropriate matrix **X** and use it to evaluate the matrix product **MX** which gives the total number of sales of each package for the month of May. [2]

(c) The prices of packages 1, 2 and 3 are \$1 000, \$1 500 and \$3 000 respectively.
The sales commission for each of the packages is 8% of the price.
This information can be represented by a row matrix **C**.

(i) Write down the matrix **C**. [1]

(ii) Evaluate the matrix product **CM**.
State what the elements of **CM** represent. [2]

9 The cumulative frequency graph below shows the distribution of marks of 300 students from School *A* in an English examination.



(a) Find the median mark. [1]

(b) Find the interquartile range. [2]

(c) The top 20% of the students obtain an A1 grade in the examination. Find the lowest mark a student must score to obtain an A1 grade. [1]

300 students from School *B* took the same examination. The table below shows the information for this school.

Median	65
Interquartile range	37

(d) Compare briefly, the results for the two schools. [2]

(e) Copy and complete the grouped frequency table of the marks scored by the 300 students in School *A*.

Marks	$0 < x \leq 20$	$20 < x \leq 40$	$40 < x \leq 60$	$60 < x \leq 80$	$80 < x \leq 100$
Frequency	15	30	55	125	75

[2]

(f) Using your grouped frequency table, calculate an estimate of

(i) the mean mark, [2]

(ii) the standard deviation. [2]

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10 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = 3 + x - \frac{1}{x^2}$.

The table below gives some values of x and the corresponding values of y.

x	0.3	0.4	0.5	0.6	0.8	1	1.3	1.6
y	-7.81	-2.85	-0.5	p	2.23	3	q	4.21

(a) Find the value of p and of q. [1]

(b) Using a scale of 1 cm to represent 0.1 unit, draw a horizontal x-axis for $0.3 \leq x \leq 1.6$.
Using a scale of 1 cm to represent 1 unit, draw a vertical y-axis for $-10 \leq y \leq 5$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the value of x in the range $0.3 \leq x \leq 1.6$ for which $x - \frac{1}{x^2} < -8$. [2]

(d) Find the x-coordinate of the point on the curve at which the gradient of the tangent is 3. [2]

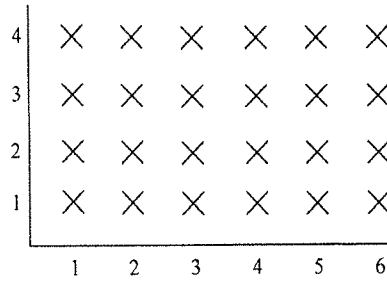
(e) **(i)** On the same axes, draw the graph of $y = -5x + 4$. [1]

(ii) Write down the x-coordinate of the point at which the two graphs intersect. [1]

(iii) Find the equation, in the form $-6x^3 + ax^2 + b = 0$, which is satisfied by the value of x found in **(e)(ii)**. [2]

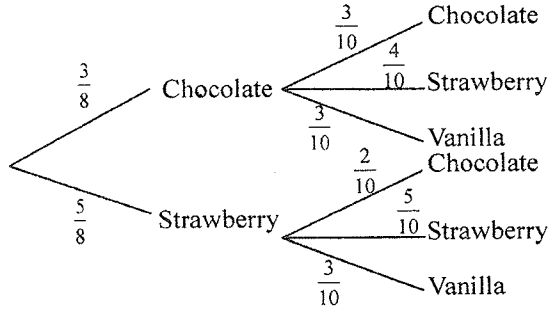
End of Paper

CEDAR GIRLS' SECONDARY SCHOOL
SECONDARY 4 MATHEMATICS
Answer Key for 2011 Preliminary Examination 2

Mathematics 4016/2			
1a	$x = \sqrt[3]{\dfrac{k-y}{\pi y+1}}$	**5bic**	$\overrightarrow{UQ} = -\dfrac{3}{2}\mathbf{a} - \dfrac{1}{2}\mathbf{b}$
1b	$\dfrac{12-5x}{2x(x-2)^2}$	**5bid**	$\overrightarrow{TQ} = -3\mathbf{a} - \mathbf{b}$
1ci	$\left(x-\dfrac{3}{8}\right)^2 - 5\dfrac{9}{64}$	**5bii**	$\overrightarrow{UQ} = \dfrac{1}{2}\overrightarrow{TQ}$. T, U and Q are collinear U is the midpoint of TQ
1cii	$x = 2.64$, $x = -1.89$ (2 d.p.)	**5biiia**	$\dfrac{3}{4}$
2a	\$10 000	**5biiib**	$\dfrac{1}{12}$
2b	$x = 7$	**6ai**	$\angle AOE = 122°$
2c	Total amount after interest from finance company = \$9 073.80 Yes. He should take up the loan from the finance company as the interest is lower.	**6aii**	$\angle AFE = 61°$
		6aiii	$\angle AEO = 29°$
		6aiv	$\angle AED = 61°$
		6b	Yes. $\angle EDC = \angle AED$ (alternate angles, AE and DC are parallel).
3ai	$MA = 11.2$ m	**7ai**	
3aii	$\angle$ of depression = 29.2° (1 d.p.)		
3aiii	$BC = 25.7$ m		
3aiv	116 m^2		
3b	51.0° (1 d.p.)		
4a	No. of litres of petrol = $\dfrac{400}{x}$		
4b	No. of litres of petrol = $\dfrac{400}{x+2}$		
4c	$\dfrac{400}{x} - \dfrac{400}{x+2} = 5$	**7aiia**	$\dfrac{1}{6}$
4d	$x = 11.69$, $x = -13.69$ (2 d.p.)		
4e	No. of litres of petrol = 7.31 l	**7aiib**	$\dfrac{1}{2}$
5ai	11.7 units	**7bi**	On next page
5aii	$-\dfrac{5}{3}$	**7biia**	$\dfrac{17}{40}$
5aiii	$B(-2,-3)$	**7biib**	$\dfrac{37}{80}$
5bia	$\overrightarrow{RS} = 2\mathbf{a}$	**8ai**	$\mathbf{J} = \begin{pmatrix} 9 & 14 & 15 \\ 13 & 10 & 9 \\ 12 & 11 & 10 \end{pmatrix}$
5bib	$\overrightarrow{RP} = 2\mathbf{a} + 2\mathbf{b}$		

8aii	$\mathbf{J}-\mathbf{M}=\begin{pmatrix}4 & 6 & 9\\ 6 & 6 & 4\\ 8 & 8 & 5\end{pmatrix}$ The elements represent the difference in the number of sales by each salesperson between the months of June and May for each package.	**9c**	81 marks
		9d	Pupils from school A performed better as their median is higher. The results of pupils from school A are more consistent as the interquartile range is lower.
8b	$\mathbf{X}=\begin{pmatrix}1\\1\\1\end{pmatrix}$, $\mathbf{MX}=\begin{pmatrix}19\\16\\12\end{pmatrix}$	**9fi**	Mean = 64.3 marks
		9fii	Standard Deviation = 21.9 marks
		10a	$p=0.82,\ q=3.71$
8ci	$\mathbf{C}=\begin{pmatrix}80 & 120 & 240\end{pmatrix}$	**10c**	Draw $y=-5$ $0.3\le x<0.35\ (\pm 0.02)$
8cii	$\mathbf{CM}=\begin{pmatrix}2200 & 1840 & 2280\end{pmatrix}$ The elements represent the sales commission each salesperson received in the month of May.	**10d**	$x=1$
		10eii	$x=0.61\ (\pm 0.02)$
		10eiii	$-6x^3+x^2+1=0$
9a	70 marks		
9b	27 marks		

7bi



9e

Marks	$0<x\le 20$	$20<x\le 40$	$40<x\le 60$	$60<x\le 80$	$80<x\le 100$
Frequency	15	30	55	125	75

10b